

# Analysis of the Established Egyptian National Gravity

## Standardization Network of 1997 (ENGSN97)



BY

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تلعب قيم واتجاهات الجاذبية الأرضية دورا مهما في مجال علوم الأرض وخاصة الجيوديسيا والجيوديسيا الطبيعية وفيزياء الأرض. من أهم أهداف علم الجيوديسيا تحديد شكل الأرض وكذلك مجال الجاذبية لها. لذا يهتم العلماء بوضع نموذج رياضي لمجال الجاذبية حتى يتسنى لهم استنتاج قيمه المختلفة من الارصاد المأخوذة . مع النمو السريع في استخدام اجهزة ال GPS في الاعمال الجيوديسية والمساحية وتحديد الارتفاعات الجيوديسية بدقة عالية بهذه الاجهزة فقد تطلب ذلك معرفة نماذج دقيقة امجال الجاذبية (الجيويد) وذلك لتحويل الارتفاعات الجيوديسية الغير مطلوبة عمليا الى ارتفاعات ارثومترية (منسوبة للجيويد) مطلوبة عمليا. ولشبكات الجاذبية اهمية كبيرة في تطبيقات قومية وعالمية عديدة . شبكة الجاذبية المسماه بنظام بوتسدام مثال للشبكات الدولية من عام ١٩٠٩ الى عام ١٩٧١ . كانت الشبكة القومية للجاذبية (NGSBN-77) الهيكل الاساسي في النصف الثاني من القرن العشرين في مصر لهيئة المساحة المصرية ايضا مجهوداتها في قياسات الجاذبية وذلك لتصحيح ارصاد الميزانيات الخاصة بها لكن كل هذه الارصاد للجاذبية لاتكفي لمتطلبات اليوم وذلك من وجهة نظر الدقة والكثافة وكذلك لقدت معظم هذه النقاط. لذا فمن الضروري جدا انشاء شبكة جديدة تستجيب لمتطلبات اليوم والتطبيقات الحديثة . هذا البحث يشرح عملية انشاء هذه الشبكة الحديثة وبيان ارصادها وشرح لخطوات معالجتها وضبطها.

### Abstract

Gravity data find multiple demands basically in two major fields of geosciences: geodesy and geophysics. The principle task of geodesy is the determination of the figure of the earth, that is extended to the determination of the exterior gravity field. The gravity field has to be modeled in order to derive geometrically defined quantities from the observations. Local gravity field representations are required for establishing geodetic control networks in geodetic and engineering surveys. With the rapid growth of

the use of the satellite-based Global Positioning System (GPS), high resolution gravity field data are needed to transform the ellipsoidal heights into orthometric heights. In geophysics, gravity data are used in a wide range of applications, such as exploration of mineral and underground water resources, monitoring crustal movements, and the study of the orbits of natural and artificial celestial bodies.

Gravity control networks are essential to support several applications, on a national and international scales. The Potsdam gravity system provides an example of an international gravity datum, from 1909 to 1971. In Egypt, the National Gravity Standard Base Net (NGSBN-77) afforded the gravity framework in the second half of the twentieth century. In addition, the Egyptian Survey Authority (ESA) usually is carrying out gravity measurements, for some specific applications, especially for correcting precise levelling observations. However, all such gravity measurements, can not satisfy the modern precise applications in Egypt. Consequently, the establishment of a precise new national gravity base network becomes an essential and urgent task for the geodetic community in Egypt, and this is actually the main point of our interest in the current investigation.

A project for re-calibration and updating the old gravity networks, and initiating a new Egyptian national gravity network was carried out. Such network will be named as the Egyptian National Gravity Standardization Network of 1997 (ENGSN97). The ENGSN97 network serves as the precise national gravity datum for Egypt. So, the ENGSN97 will be documented in details in this research.

## **1- Introduction**

The earliest absolute gravity observations in Egypt have been carried out in 1908 at Helwan observatory. This station was considered as the fundamental gravity station in Egypt with a gravity value of 979.295 Gal [Cole, 1944]. Other seven gravity stations in Egypt and Sudan have been observed in that year. These stations have been tied to Kew, London. The Egyptian Survey Authority, formally the Survey of Egypt, has started its first gravity survey in 1927 using a pendulum apparatus [ibid].

Starting from 1922, some foreign oil companies have carried out gravimetric survey as a geophysical exploration tool, mainly in the Gulf of Suez. From 1922 to 1938, more absolute gravity measurements have been carried out at Ras Gharib, Helwan, Suez, Shadwan, Rahmi, Wadi El-Natrun, and Amria. From 1938 to 1950, more precise gravimeters have been used in gravimetric surveys. Within an international gravity program, twenty-one stations have been established in Egypt in 1950-1951 and tied to the Potsdam gravity reference system [Kamel and Nakhla, 1985].

In August 1971, the International Gravity Standardization Net (IGSN-71) was introduced as the new global gravity reference system. The network contains 473 primary stations (eight of them are absolute stations). The standard errors for the net's gravity values are less than 0.1 mGal [Morelli et al., 1974]. As a part of the IGSN-71 activities, eleven gravity stations have been measured in Egypt with standard deviations range from 0.024 to 0.035 mGal.

A ten-year project (1974-1984) for compilation of gravity maps of Egypt has resulted in the National Gravity Standard Base Net (NGSBN-77). The NGSBN-77 consists of 66 stations, and has been tied to IGSN-71 stations located at Cairo International airport, Helwan observatory, Luxor, Aswan, and Port Said. The mean square errors for the observations range between 0.04 - 0.47 mGal [Kamel and Nakhla, 1985].

ESA has carried out some gravimetric surveys along first order leveling lines. concentrated in the Northern part of Egypt. ESA is usually conducting several gravity missions, needed to compute the required corrections for the first-order levelling routes, see [Nassar, 1976] and [Nassar, 1977]. Several other authorities in Egypt conduct gravity missions for special purposes. The National Research Institute of Astronomy and Geophysics, for example, has observed several small gravity nets, as a part of multi-observables geodetic networks, serving for the detection of crustal deformation. Most of these loops are concentrated in the active crustal movement zone of Aswan lake [Groten and Tealeb, 1995].

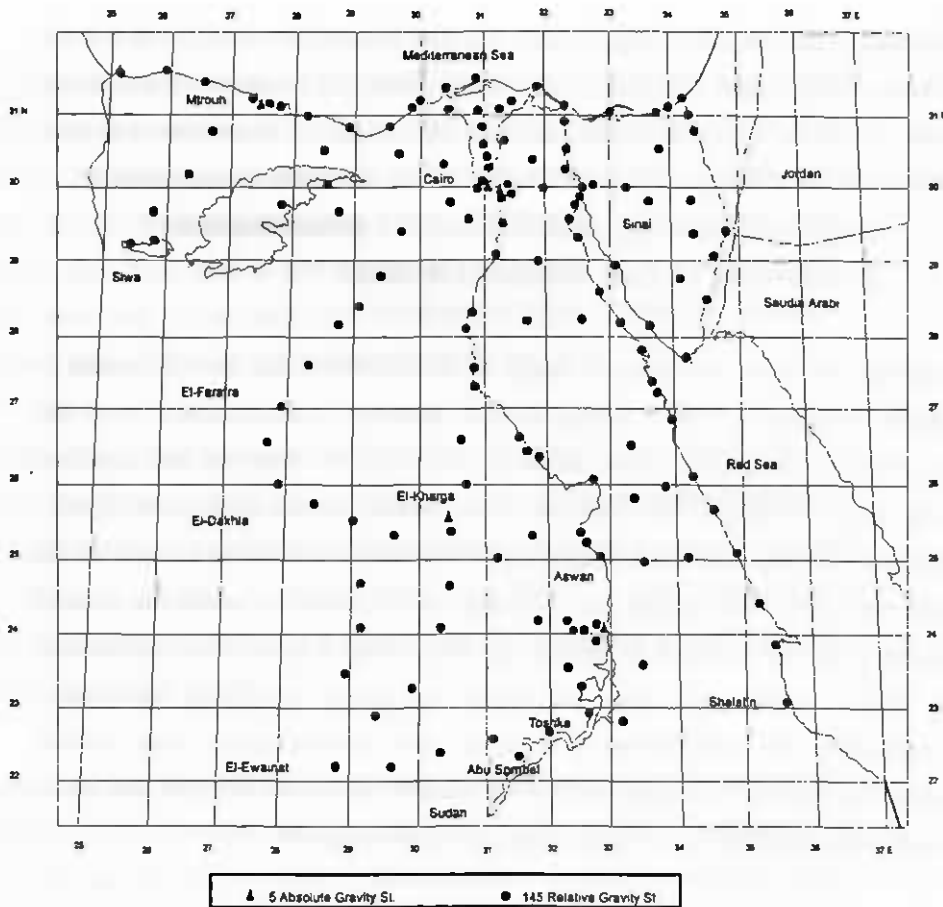
According to the ENGSN97 project's goals, the field observation campaigns include the collection of three types of measurements: relative gravity, GPS coordinates, and precise levels. This is, of course, beside the necessary absolute gravity measurements at some selected stations. Consequently, each ENGSN97 station has known precise values of:

gravity acceleration ( $g$ ), geodetic latitude ( $\phi$ ), geodetic longitude ( $\lambda$ ),  
geodetic height ( $h$ ), and orthometric ( $H$ ) height.

Regarding the above conditions, a design of the ENGSN97 has been optimized as depicted in Figure (1). Prior to starting the field campaigns, a reconnaissance survey has run and showed that, unfortunately, almost all of the NGSBN-77 stations have been lost, except about five stations only. Therefore, it was decided to re-establish the NGSBN-77 stations at the same locations as long as the station selection conditions are satisfied. In areas where new infrastructures have been built, new locations are selected as close as possible to the old locations. In addition, the reconnaissance showed also that most of the IGSN-71 stations have been lost except two stations at Helwan Observatory. Consequently, the establishment of a precise new national gravity base network becomes an essential and urgent task for the geodetic community in Egypt, and this is actually the main point of our interest in the current investigation.

## **2- Description and field measurements of ENGSN97 gravity network**

The gravity network consists of 145 relative gravity stations and 5 new absolute gravity stations established in Egypt in 1997, Figure (1). The network has been observed through 51 loops, Figure (2). The minimum, maximum, and average distance spacing between stations are 0.136, 128.144, and 65.988Km respectively. Seven LaCoste and Romberge relative gravimeters have been utilized in the observation campaigns of ENGSN97 network. Table (1) presents a summary of the ENGSN97 input statistics.



**Figure 1**

**The Egyptian National Gravity Standardization Network "ENGSN97"**

The gravimeter reading is converted to corresponding milligal values by using the calibration table of each gravimeter provided by the manufacture. It is worth mentioning that there were no trustable absolute gravity stations still existing in Egypt before 1997 to perform field calibration of the gravimeters. The solar-plus-linear tide correction, subtracted from the observations, are computed based on the formula of Longman except that the values are increased by 16% to compensate for the finite deformation or compliance of the Earth [LaCoste and Romberge, 1989b]. The GRAVPAC software was used to perform these two steps and to produce the so-called metered-gravity values which become the basic observables, to be input to the developed processing programs.

Two different observation scenarios are applied in the field campaigns in this network: the step method; and the profile method. Both of them give precise results regarding the drift control. However, the step method is economically expensive since it requires three stops over every station while in the profile method only two stops per stations are needed. The original scenario of the profile observation scheme is observing the station sequence as 1-2-3-4-4-3-2-1. Tacking the time break after the first observation over station 4 will divide the loop into two parts: 1-2-3-4 and 4-3-2-1. Both of the new data series do not have any repeated observations, which makes the drift estimation is impractical. Two alternatives are proposed:

- (1) If the loop observation time is relatively short, there is no need for the break and the observation scenario should be modified to be: 1-2-3-4-5-4-3-2-1, where station 5 is any arbitrary station, not of interest. This station is observed just to separate the two consecutive observations over the network station 4 and to avoid having gravimeter dial remain fixed.
- (2) In case of a long observation time of the loop, the observation scenario should be: 1-2-3-4-4-break-4-4-3-2-1.

GPS observations are taken to obtain accurate geodetic coordinates at all the network stations. The stations are tied to first order stations of ESA. In the same time, levelling observations are made to obtain the orthometric heights of the network stations. The network stations are connected to the nearest Bench Marks of ESA in every case.

### **3- Assessment of the developed program GNPA**

A comparison has been carried out between the results of the developed GNPA program and the GRCOMP program, for the purpose of checking and assessment of the performance of the former one. GRCOMP is a program developed by the U.S. Defense Mapping Agency (DMA) for processing field gravity data. Although GRCOMP can use gravity observations from different gravimeters in a loop, it processes the data from each instrument separately [Stizza, 1997]. GRCOMP starts with the dial readings of the gravimeters, applies the Earth tide correction, and estimate the instrument drift correction. GRCOMP is an easy-to-use program in an interactive way with the user. On the other hand, A disadvantage of GRCOMP is that it is designed to process the data

from the global-range model G gravimeters only. An advantage of GRCOMP is the utilization of the least-squares adjustment to provide standard deviations of the station gravity values as a tool for judgment the executed gravity loops in the field.

Both programs have been used to process the same loop, that includes the new absolute gravity station in Helwan (named SRI5), the triangulation station O1, and the IGSN-71 gravity station in Helwan (Cairo-B). Seven LaCoste and Romberge relative gravimeters have been used to conduct this loop. The loop consists of 63 total observations, 9 observations per gravimeter. The results show agreement, in the point gravity value between GRCOMP and GNPA programs within 0.005 mGal, after performing the least-squares adjustment [Dawod and Alnaggar, 1997].

#### **4- Methodology of gravity data analysis and network adjustment**

This part is devoted to the data processing of the ENGSN97 network, including the highlight of the developed computer programs, needed for all involved computations, as well as performing different solutions, for investigating all effecting factors on the final results, one at a time. The efficient newly-developed processing models have been utilized. Several LF90-language computer programs have been developed to process, adjust, and analyze gravity networks in several stages as follows:

##### **4.1 Primary analysis for each loop using one gravimeter at a time**

The first program processes data of one loop conducted by one gravimeter utilizing the equation:

$$r_j = g_j - O + \Delta t_{ij} \cdot d \quad ( 1 )$$

where,

$\Delta t_{ij}$  is the difference in time between the running station j and the initial fixed station i,  
d is the drift of the used gravity meter ( assuming linear function ).

This is the basic equation describing the mathematical relationship between the gravimeter readings as observables, and the gravity values of the observed stations, taking the systematic drift error of the gravity meter, as well as the unknown gravity

orientation parameter (O) at the fixed station i, into considerations. Every gravimeter reading on any station in the observed gravity loop gives one equation of that form.

Note that, in the above observation equation, the only observed quantity is the gravimeter reading,  $r_j$ , whereas the unknown parameters will be three quantities,  $g_j$ , O, and d. In other words, for "n" observed gravity stations, there will be "n" observation equations, and the number of unknowns will be two plus the number of unknown gravity stations. Therefore, if "n" equals the number of unknowns, there will be one solution only for the system of observation equations. However, this is not the case in practice, where there should be as many redundant observations as possible. In such a case, one will be faced with an over-determined mathematical model, whose solution becomes possible on the basis of least-squares principle, see [Nassar, 1981].

The objective of this program is to analyze each gravimeter's observation separately to investigate the quality and the performance of each gravimeter in a loop. The data input of this developed program include:

- \* the gravimeter dial readings (in mGal units),
- \* the corresponding time of observations (in hours),
- \* the apriori variance factor to be used in the measurement weighting process,
- \* the absolute gravity value of at least one station to be held fixed in the adjustment,
- \* an alphabetic stations names and gravimeters numbers to be tabulated in the output.

The program output contains:

- \* the estimated gravity value for each gravity station along with its estimated standard deviation,
- \* the estimated aposteriori variance factor,
- \* the estimated gravimeter drift coefficient with its estimated standard deviation,
- \* the estimated gravity orientation parameter with its estimated standard deviation.



## 4.2 Analysis of each loop using different gravimeters combined together

The second developed program handles the gravity data of all gravimeters used in observing a loop. Therefore, the unknowns to be estimated are the gravity values of all stations beside a gravity orientation and a drift unknowns for each virtual gravimeter. This program has a built-in outlier detection subroutine used to flag the erroneous observations based on the results of the  $\tau$  statistical test. The corresponding observation equation model is read as follows:

$$r_{jk} = g_j - O_k + \Delta t_{i,j,k} \cdot d_k \quad ( 2 )$$

where,

$k$  is the gravity meter number,

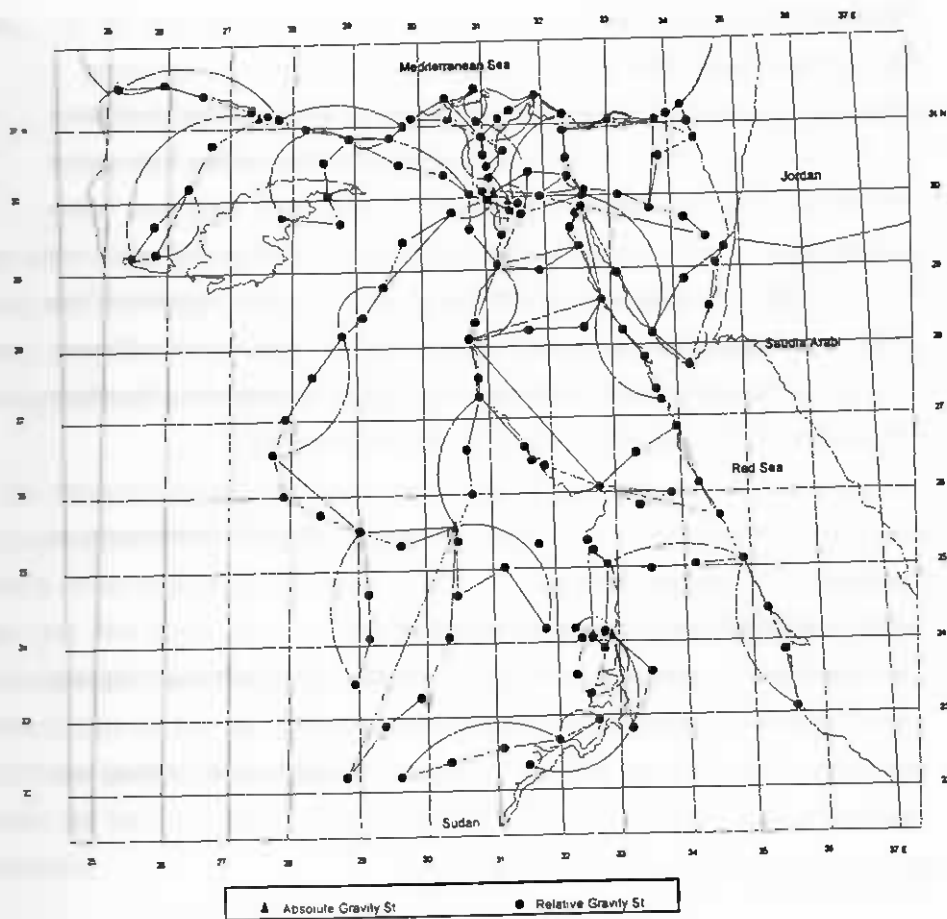
$r_{jk}$  is the reading of the  $k^{\text{th}}$  gravity meter on station  $j$ ,

$\Delta t_{i,j,k}$  is the difference in time between the running station  $j$  and the initial fixed station  $i$ , using the  $k^{\text{th}}$  gravity meter,

$d_k$  is the drift of the  $k^{\text{th}}$  gravity meter, and the remaining symbols are the same as defined before.

In this case, there will be "n" observation equations of that type, where the number of unknown parameters will be equal to the number of unknown gravity stations "j", and twice the number of the used gravity meters "k". The data input of this developed program includes:

- \* the virtual gravimeter dial readings (in mGal units), i.e., a change in the gravimeter number means either an occurred drift or a new gravimeter data will be followed,
- \* the corresponding time of observations (in hours),
- \* the apriori variance factor to be used in the measurement weighting process,
- \* the absolute gravity value of at least one station to be held fixed in the adjustment, and
- \* an alphabetic stations names and virtual gravimeters numbers to be tabulated in the output.



**Figure 2**  
**ENGSN97 Used Gravity Loops**

The program output contains:

- \* the estimated gravity value for each gravity station along with its estimated standard deviation,
- \* the estimated a posteriori variance factor,
- \* the estimated drift coefficient for each virtual gravimeter with its estimated standard deviation,
- \* the estimated gravity orientation parameter for each virtual data series with its estimated standard deviation,
- \* the estimated residuals with their estimated standard deviation.
- \* the critical tau value corresponding to the computed degrees of freedom,

- \* the normalized residuals, and
- \* A list of the flagged outliers.

When dealing with gross-error detection, it was decided to delete only the observation that had the largest normalized residual. Most of the literature dealing with outlier detection stressed that any algorithm should not be used as a black box which automatically cleans the observations. This warning may be explained when we remember the original two assumptions behind the Tau-test for outlier detection [Chen et al, 1987]: (1) all observations are normally distributed; and more important (2) only one outlier is assumed to be present in the data set, at a time. Consequently, the whole theory breaks down if the observations include two or more gross errors.

The following rational approach is followed in order to obtain as much correct results as possible [Alnaggar and Dawod, 1995a]:

- \* Initial adjustment is carried out using all observations.
- \* If more than one normalized residual exceed the critical  $\tau$  value, only the observation with the largest normalized residual is deleted.
- \* Adjustment is repeated again with  $n-1$  observations leading to new residuals and new  $\sigma_o^2$
- \* This process is repeated until all outliers are flagged.

#### 4.3 The processing of the entire ENGSN97 gravity network

The third developed computer program, GNPA: Gravity Network Processing and Adjustment, is the main computational tool used in this research study. It processes and adjusts a gravity network that consists of several field loops observed by several gravimeters using the newly-developed model where the observation equations model will be:

$$r_{j,k,l} = g_j - O_{k,l} + Z_{j,l,k} \cdot c_{k,l} + \Delta t_{l,j,k,l} \cdot d_{k,l} \quad (3)$$

where,

$Z_{j,k,l}$  is the original dial reading ( in counter units) of this observation  $j$  in the  $l^{\text{th}}$  data series using the  $k^{\text{th}}$  gravimeter, and

$\epsilon_k$  is the unknown error in the used linear calibration coefficient for the  $k^{\text{th}}$  gravimeter, that was used in transforming the dial reading in counter units into the equivalent mGal units, as estimated from the gravimeter readings on gravity stations with known absolute values.

In addition, the symbol "l" in the equation represents either the data set number before and after each break in the same loop, or indicates the data set number for each added new observed loop. By this way, one can expect more accurate estimation for all the unknown parameters, especially the gravity values at the unknown gravity stations.

The unknowns contain the gravity values of observed stations and two unknowns (orientation and drift) for each "virtual instrument". If a loop contains a break, its data set is further divided into two virtual data series and two unknowns have to be estimated to each data series. This situation resembles the case as if two different "virtual instruments" have been used in this loop. Regarding the gravimeter drift estimation, this implies the fact that during the break time a zero static drift is assigned and two different dynamic drift coefficients are to be estimated for the two data series. The data input and the output results of this developed program are the same as in the previous program.

## **5- Performed different solutions to obtain the best optimum results for the ENGSN97 network**

In this section, the developed program GNPA has been run to adjust the entire ENGSN97 gravity network. Six solutions are made as follows:

### **5.1 First solution: Introducing one absolute gravity station as completely fixed in the adjustment of the network**

The purpose of this solution is to investigate the consistency between the relative gravity measurements, taken by both G and D models of LCR gravimeters, and those absolute gravity measurements, taken at five stations only by the FG5 absolute free-fall device. In this case, one absolute gravity station only is held fixed, which is taken in our case here

as the SRI5 station at Helwan, as the nearest absolute station to the center of gravity of the entire ENGSN97 network. In other words, the remaining four absolute stations will be retained aside, as if they were not absolutely measured at all. Following the same computational approach, of assigning very large weights for absolute station to be held fixed, the corresponding diagonal element of the normal-equation matrix pertaining to SRI5, will be assigned a very large value. In this case, an estimate of the gravity value at each of the four absolute stations will be obtained after the least-squares adjustment process, which gives one a chance to be compared against the already-known absolute value at the same station. The analysis of the difference between the two sets of gravity values, for those four absolute stations, will indicate a good idea about the consistency between the relative and absolute gravity measurements performed in our network here. Of course, such an adjustment follows the ordinary known minimal-constrained approach of the least-squares adjustment.

The differences between the known absolute gravity values, and the corresponding estimated values from the above adjustment, for the four absolute stations mentioned above are computed. Recall that, the assigned precision, in terms of standard deviations, for all relative gravity measurements were taken as 0.030 mGal. The above mentioned differences are almost in the same order of the precision of the relative gravity measurements, which indicates the existing consistency between both types of gravity measurements, namely the absolute and relative measurements.

## **5.2 Second solution: Introducing appropriate weights for the absolute gravity stations**

Following the same approach of treating the absolute gravity stations, as quasi-observables or weighted parameters, with their appropriate estimated standard deviations, as obtained from the absolute measurement technique, the residual equation for each one of the five absolute stations will be:

$$V_i(\text{abs}) = g_i(\text{to be estimated}) - g_i(\text{absolute}), \quad i = 1, 2, 3, 4, \text{ and } 5$$

This means that each value of the absolute gravity station will receive a certain correction or residual  $V_i(\text{abs})$ , after the adjustment. Of course, all those five pseudo-

observations or residual equations, will be added to the ordinary system of observation equations for relative gravity measurements, while the whole system is combined together and will be solved simultaneously

The differences between those estimated values, from the combined adjustment, and the corresponding known absolute values, as obtained from direct absolute measurements are computed. The differences between the estimated gravity values and the known absolute values are almost zero value. This means that, the treatment of the absolute gravity stations as quasi-observables, gives almost the same results as if they were treated as fixed quantities.

### **5.3 Third solution: Investigating non-linear drift against linear drift functions**

In this solution, the validity of taking the drift function of the used relative gravimeters as non-linear instead of the usual linear function, will be investigated, whether it could improve the quality of the network or not. A non-linear drift model has been tried for all loops. Therefore, the term on the right-hand side of the gravimeter reading observation equation, say in the general model given by equation (3), becomes  $(d_1 \Delta t + d_2 \Delta t^2)$  in which two unknown coefficients  $d_1$  and  $d_2$  are needed to be estimated, instead of only one coefficient in case of linear drift function.

The entire ENGSN97 network was adjusted, again, using the second solution above, after introducing the non-linear drift function. After the least-squares adjustment, an estimated value for the second drift coefficient  $d_2$  was obtained, along with its estimated standard deviation, as one new unknown parameter. In addition, the estimated value for gravity station and their covariance matrix, are obtained as the main output results. The obtained results show the following three remarks:

- The value of the second term  $d_2 \Delta t^2$ , of the non-linear drift function contribution, is relatively very small compared to the first part  $d_1 \Delta t$ , which in most of the cases does not exceed the 5% level.

- \* The estimated standard deviation for the drift coefficient  $d_2$  exceed, in most of the cases, the value of the coefficient itself, which means that the drift coefficient  $d_2$  is statistically insignificant, that is it can not be distinguish from the zero value, from the statistical point of view.
- \* The adjusted point gravity values of the network attain relatively large values for their standard deviations, at a number of stations, when compared to the corresponding good estimate obtained from the second solution above.

Based on the above obtained results and remarks, it can be concluded that the second part of the non-linear drift function is statistically insignificant, and should be neglected, particularly since it deteriorates the overall quality of the entire gravity network by about 18% in our network here. In other words, the linear drift function for LCR gravimeters will be the best to be used instead of any suggested non-linear functions.

#### **5.4 Forth solution: Introducing different weights for different gravimeters**

Several gravity surveys show that the precision of the D model of LCR gravimeters is better than that of the G model gravimeters [Torge, 1989a]. In the previous solutions, all relative observations were assigned a standard deviation of 0.03 mGal, that is equal weights for both G and D LCR gravimeter models. In the present solution, the entire ENGSN97 network was adjusted, again, using a better precision (standard deviation of 0.02 mGal) for the observations carried out by the D model of the relative gravimeters. This data series contains 248 observations in 37 virtual loops. The rest of the observations were assigned a standard deviation of 0.03 mGal, for the G model gravimeters.

The obtained results, in terms of the estimated standard deviations of the point gravity values of the entire ENGSN97 network stations, indicated that there is a slight improvement in the precision (in the order of approximately 13%), as compared to those obtained from the second solution above, when equal weights were used. This may be due the fact that the number of the observations of the D-model gravimeters is slightly

small compared with the total observations (23% approximately). Such slight improvement has occurred especially for the stations observed by the d model gravimeters in addition to the original G gravimeters.

### **5.5 Fifth solution: Introducing different weights for loops according to their observation times**

Although a gravity loop must not exceed seventy two hours as observation time, it is the author's experience from the field campaigns that the longer the time span of a loop, the more problems encountered in the field regarding the observation circumstances. Tare (unexpected jump) is an example of these sudden problems that are quite difficult to be modeled. In order to avoid these uncertainties, a worse precision (standard deviation of 0.050 mGal ) was assigned to all observations taking along the loops, which their observation have been continued for more than one-day. These observations are found to be 228 in 12 virtual loops. The remaining observations of the rest of the loops, observed over a time span less than one day, were assigned the pre-specified standard deviation of 0.030 mGal, which amounted to 857 observations over 39 virtual loops.

The obtained results of the estimated gravity values, indicated that there is slight improvements in their precision compared to the corresponding values from the standard adjustment of the second solution, in the order of approximately 18%.

### **5.6 Sixth solution: After satisfying all significant-affecting criteria**

Based on the obtained results from the above discussed five solutions, it can be concluded that the best appropriate way of adjusting the entire ENGSN97 gravity network, will be performed with the following items taken into consideration:

- The five absolute gravity stations should not be treated as purely fixed, but taken as quasi-observables with appropriate large weights.
- Each used gravimeter should be assigned different weight for all observations taken by it, depending upon the reported precision of the manufacturer and by previous



investigators, where a linear drift function should be used for all LCR gravimeter models.

- \* For all loops observed over span times exceeding the one-day limit, should be assigned less weights for their encountered observations, as opposed to those loops completed in a period less than one day.

After satisfying all the significantly influence factors, another adjustment of the network has been performed, which is named as solution number six. However, such adjustment included all the 1085 gravimeter readings taken over the 51 virtual loops, as collected by using five actual G-model gravimeters and two D-model gravimeters of the LCR type. Of course, one should assume a hypothesis of possible outlier observations, existing for some of those collected gravimeter readings. In order to have meaningful-final results, based on good-quality observations, all such erroneous readings must be filtered out from the system, and a best solution is obtained using all the remaining cleaned observations, as will be given in the next sub-section.

### **5.7 Final solution: After filtering out all outlier observations**

Filtering out the observations from existing outliers can be simply performed, using the appropriate tau test for detecting outliers, which is based on testing the normalized residuals against a critical tau value, instead of testing the pure residuals themselves.

Each estimated residual  $V_i$  of a gravimeter reading  $r_i$ , should be normalized first by dividing its value by its corresponding standard deviation. The critical value of the tau statistic is then computed, based on the degree of freedom and the probability level using the student t-distribution function. If the normalized residual  $T_i$  exceeds the computed critical limit, the corresponding observation is suspected to contain some sort of gross errors, and hence, should be rejected from the system. The least-squares adjustment is repeated again using the remaining observations, after rejecting the outliers, and a new set of estimated residuals can be obtained, and the tau test is performed again, and repeating the process until all measurements are cleaned out.

For the ENGSN97 gravity network, in our hands here, eight consecutive solutions were repeated, for the purpose of detecting and rejecting outliers, until no more outliers are flagged. A total of 44 observations out of the original 1085 observations have been flagged and removed. These observations constitute only about 4 % of the total number of the measurements, which is another indication of the goodness of the ENGSN97 network. The solution of adjusting the network, as performed free from all the rejected 44 outliers, represents the final best-optimized solution for our ENGSN97 network.

Comparing the estimated standard deviations of the final solution, with the corresponding values as obtained from the sixth solution, one can notice significant improvement in the overall precision of the finally adjusted network. In other words, the removal of all existing outliers from the set of gravimeter readings, will certainly improve the overall quality of the final solution of the network. Such improvement amounts to approximately 37%.

## **6- Essential characteristics of the final solution of the ENGSN97 network**

The most essential information of the final adopted solution of the ENGSN97 network are summarized in Table (1).

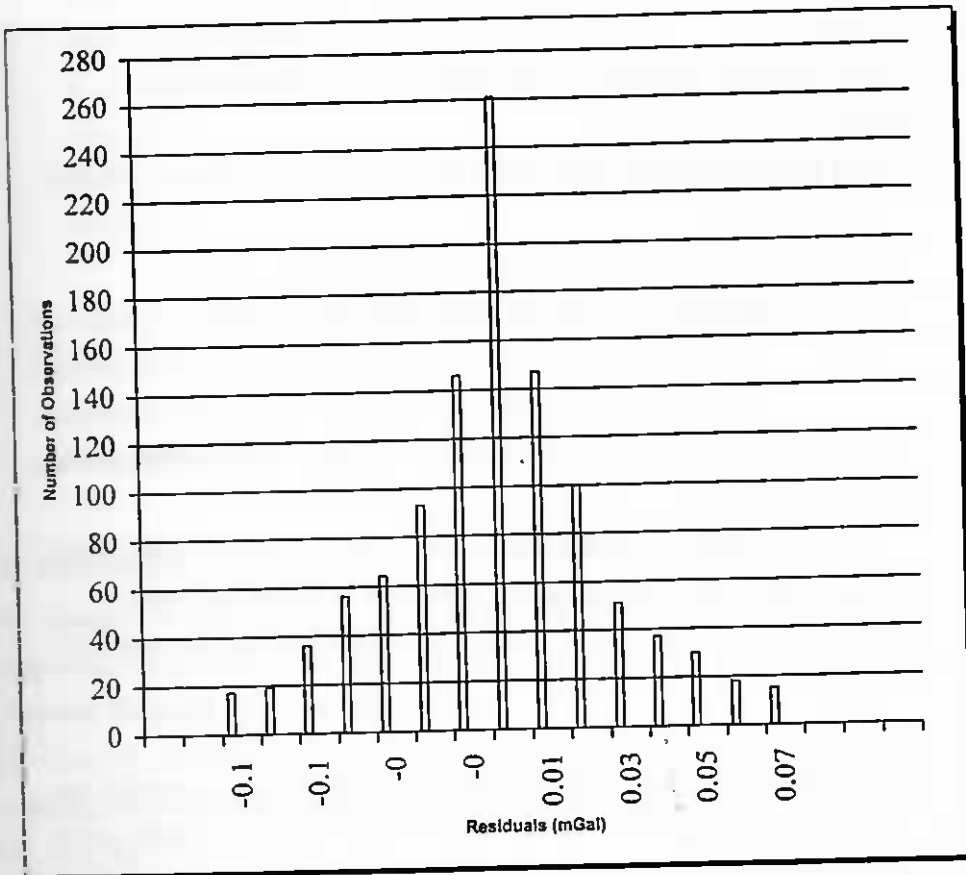
It may be interesting to investigate the distribution of the residuals, of the 1045 cleaned gravimeter readings, as estimated from the final solution of the network, which are depicted in Figure 3. The general trend of such a distribution, approximately follows the ordinary Gaussian normal distribution curve. This indicates that the remaining residuals of the cleaned gravimeter readings, are representing random errors only, with their mean value approaching the statistical mean value of zero. In other words, such cleaned used gravimeter readings are not affected by any sort of systematic errors or biases, associated with the used instruments, and used observational techniques and computations.

It is worth to mention here that, after finishing the data processing and the final adjustment solution of the ENGSN97 network, the obtained results are checked out at the Technical University in Graz (TU-Graz ). Austria. The data of ENGSN97 have been re-processed and re-adjusted again on the university mainframe computer using the classified computer programs available in the physical geodesy department of TU-Graz.

The results show an agreement in the order of few microGals between the final solution and that solution of TU-Graz [Sünkel, 1998]. It means that, this check proves that all processing and adjustment stages carried out, for ENGSN97 are of high quality and reliability and emphasis that ENGSN97 has a high-level of precision as a gravity datum for Egypt. This has been supported by the performed re-adjustment of the network at TU-Graz, using different independent computer packages.

**Table 1**  
**Essential Information For The Final Adjustment**  
**Of The ENGSN97 Network**

Number of stations	150	
Number of observations	1045	
Number of loops	51	
Minimum station separation	0.136	Km
Maximum station separation	128.144	Km
Average station separation	65.988	Km
Number of actual gravimeters	7	
Number of virtual gravimeters	133	
Number of unknowns	408	
Number of degrees of freedom	637	
Minimum standard deviation of gravity values	0.002	mGal
Maximum standard deviation of gravity values	0.048	mGal
Minimum adjusted gravity value	978679.776	mGal
Maximum adjusted gravity value	979405.981	mGal
Gravity range over Egypt	825.205	mGal



**Figure 3**  
**Frequency Distribution of the Residuals of the**  
**Gravimeter Readings Over the ENGSN97 Network**

## 7- Conclusions

Based on the above-mentioned processing and adjustment trials in this study, The final solution of the ENGSN97 has been processed according to the following processing specifications:

- Using weighted parameter approach in least-squares adjustment for the five absolute gravity stations,
- Using 0.02 mGal standard deviation for the D-gravimeter relative gravity observations,

- Using 0.03 mGal standard deviation for the G-gravimeter relative gravity observations,
- Using standard deviation of 0.05 mGal for long gravity loops completed over one day period.
- Using standard deviation of 0.03 mGal for loops completed within a period of less than one day,
- Using linear-drift model for LCR relative gravimeters, and
- Applying outlier detection routine for cleaning the data from gross errors.

The obtained results from the final solution and adjustment of ENGSN97 gravity network indicated that:

- the gravity field over the Egyptian territory, expressed in terms of the gravity acceleration value  $g$  ranges between 978679.776 and 979405.981 mGal
- the Standard Deviation of the gravity values ranges between 0.002 and 0.048 mGal

Consequently it can be concluded that the present Egyptian gravity network can be categorized as first order gravity network, with about 150 gravity points evenly distributed across the Egyptian territory. Such present gravity network can be considered to be more reliable, more gravity stations, and strengthened with addition precise positional information, namely horizontal position in terms of Latitude and Longitude, as well as orthometric height, as compared to the previous national gravity network (NGSBN-77)

In the context of establishing and adjusting the new National Gravity Standardization Net (ENGSN-97), the following information can be considered as great achievement and wealth information that can be added to the Egyptian geodetic data bank, which includes:

- Five absolute gravity stations

- 145 relative gravity stations with almost the same accuracy of the absolute ones, and they are well distributed in the country
- at all 150 stations, the geodetic coordinates are determined precisely using GPS
- at all 150 stations, the orthometric heights are determined using precise levelling

Such geodetic data will be essentially needed for future updating of the Egyptian geoid, which will be in turn used for related geodetic network computation and adjustment, as well as associated mapping and other geodetic applications.

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